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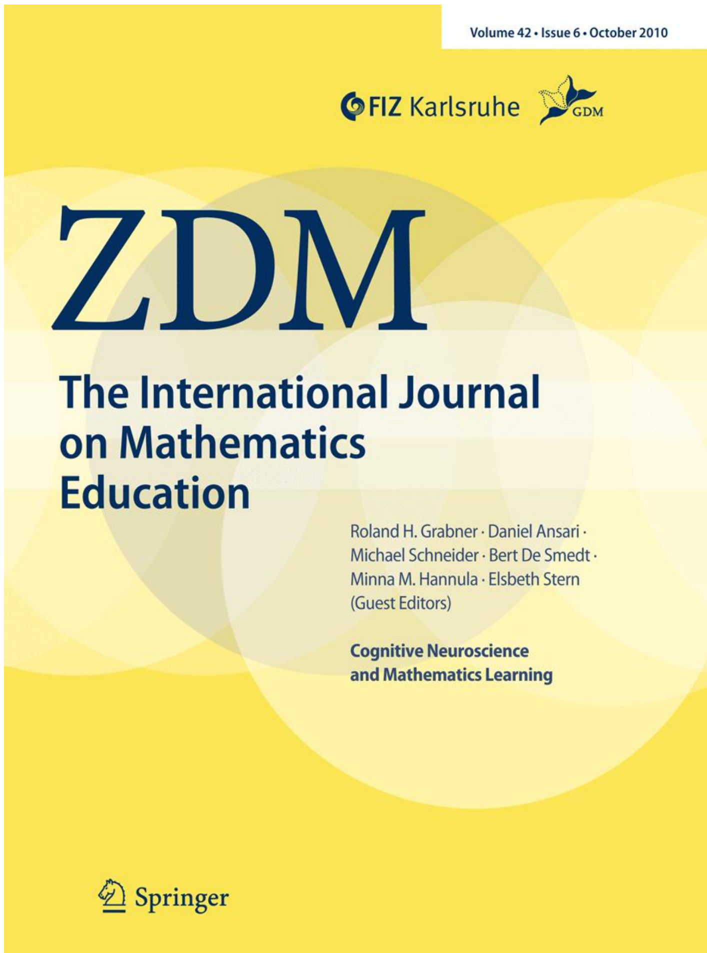

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ORIGINAL ARTICLE

Developmental cognitive neuroscience of arithmetic: implications for learning and education

Vinod Menon

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Abstract In this article, we review the brain and cognitive processes underlying the development of arithmetic skills. This review focuses primarily on the development of arithmetic skills in children, but it also summarizes relevant findings from adults for which a larger body of research currently exists. We integrate relevant findings and theories from experimental psychology and cognitive neuroscience. We describe the functional neuroanatomy of cognitive processes that influence and facilitate arithmetic skill development, including calculation, retrieval, strategy use, decision making, as well as working memory and attention. Building on recent findings from functional brain imaging studies, we describe the role of distributed brain regions in the development of mathematical skills. We highlight neurodevelopmental models that go beyond the parietal cortex role in basic number processing, in favor of multiple neural systems and pathways involved in mathematical information processing. From this viewpoint, we outline areas for future study that may help to bridge the gap between the cognitive neuroscience of arithmetic skill development and educational practice.

1 Aims and scope

Mathematical skills are arguably one of the most important cognitive abilities that a child must master. What are the changes that occur in the brain as children begin to develop more complex and quantitative ways of thinking? Why do children show marked individual differences in mathematical abilities, and what factors contribute to these differences? These questions have fueled the work of developmental and education psychologists for decades (Dowker, 2005; Geary, 1994; Geary, Hoard, & Royer, 2002; Siegler, 1998; Siegler & Stern, 1998). Now, with advancements in quantitative brain imaging and the use of targeted cognitive experiments, we are uniquely positioned to answer these questions.

Arithmetic skills build on a core number knowledge system, for representing numerical quantity using abstract symbols, which is typically in place by the age of 5 years (Barth, La Mont, Lipton, & Spelke, 2005). Neural mechanisms underlying the development of these core numerical systems are reviewed elsewhere (Ansari, 2008); here, we focus on the development of brain systems involved in arithmetic. The approach taken here is to highlight major findings related to key component processes involved in arithmetic problem solving and reasoning. We first review core cognitive and brain processes involved in arithmetic processing and discuss the implications of relevant studies in adults for understanding the neural basis of arithmetic skill development. Recent studies have focused on various aspects of arithmetic processing, including (1) retrieval, (2) computation, (3) reasoning and decision making about arithmetic relations, and (4) resolving interference between multiple competing solutions (interference resolution). They help to clarify which brain areas are critically and consistently engaged during arithmetic tasks, which regions provide a supportive role in arithmetic, and which brain areas contribute to arithmetic learning. We then discuss recent brain imaging studies of arithmetic in children and examine how they inform our understanding of skill development. We also highlight areas for future study

V. Menon (✉)
Symbolic Systems Program, Program in Neuroscience, Department of Psychiatry & Behavioral Sciences, and Department of Neurology & Neurological Sciences, Stanford University School of Medicine, Stanford, CA 94305-5778, USA
e-mail: menon@stanford.edu

that will help bridge the gap between cognitive neuroscience and educational practice in this domain.

2 Cognitive neuroscience of mental arithmetic

Findings from lesion and imaging studies, mainly in adults, provide a conceptual framework for initial studies of arithmetic in children when considered alongside developmental theories. In this section, we summarize the relevant findings from lesion and brain imaging studies in adults and discuss their implications for developmental studies of arithmetic.

We first clarify some of the key cognitive processes contributing to accurate arithmetic task performance (Fig. 1). Comprehension of numerical properties (i.e., number magnitude and cardinality) can be considered the basic building block from which arithmetic is constructed. Beyond this foundation, fact retrieval and calculation are two core functions mediating arithmetic proficiency. Memory retrieval based on prior learning allows for fast access of learned arithmetic facts. Working memory resources (i.e., temporary storage and manipulation of information) are needed when results cannot be easily retrieved and need to be calculated based on decomposition and other rules. In conjunction with these memory processes, attentional resources, sequencing mental operations and decision making also influence the speed and accuracy of performance. These domain-general cognitive resources are as vital as core numerical knowledge in classroom settings when a child is learning to improve arithmetic skills. In this article, we address the contributions of both core and auxiliary components underlying arithmetic. An important aspect of this inquiry relates to how the role of these auxiliary processes changes with the maturation of problem-solving skills (Fig. 2) in children and adolescents.

Deficits in the posterior parietal cortex (Fig. 3) are classically thought to underlie dyscalculia, a disorder of numerical competence and arithmetic skill, which is manifested in individuals of normal intelligence who do not have acquired neurological injuries (Temple, 2002). Within the posterior parietal cortex, the intra-parietal sulcus, angular gyrus, supramarginal gyrus and perisylvian cortex have all been implicated in acalculia. Acalculia has also been reported in patients with lesions to the prefrontal cortex (Besnon & Weir, 1972; Henschen, 1920; McCarthy & Warrington, 1988; Takayama, Sugishita, Akiguchi, & Kimura, 1994; Warrington, 1982). A number of functional dissociations between brain regions differentially involved in specific operations such as addition, subtraction and multiplication have been suggested in literature (Chochon, Cohen, van de Moortele, & Dehaene, 1999; McNeil & Warrington, 1994; van Harskamp & Cipolotti, 2001). Dissociations have also been reported between retrieval and calculation. For example, one case study reported a patient who experienced deficits in complex calculation while maintaining preserved simple arithmetic fact retrieval after lesions of the left posterior parietal cortex (Delazer & Benke, 1997). While lesions in the posterior parietal cortex often have dramatic consequences for mathematical information processing, they appear to be variable in the specific type of arithmetic deficits that are seen across individual patients (Kahn & Whitaker, 1991; McCloskey, Harley, & Sokol, 1991). In spite of an array of dissociations reported in literature, lesion studies have lacked adequate anatomical specificity and have yielded limited knowledge about the functional role of specific brain regions in arithmetic.

Functional magnetic resonance imaging studies have provided more detailed and precise localization of posterior parietal cortex and other distributed brain regions involved

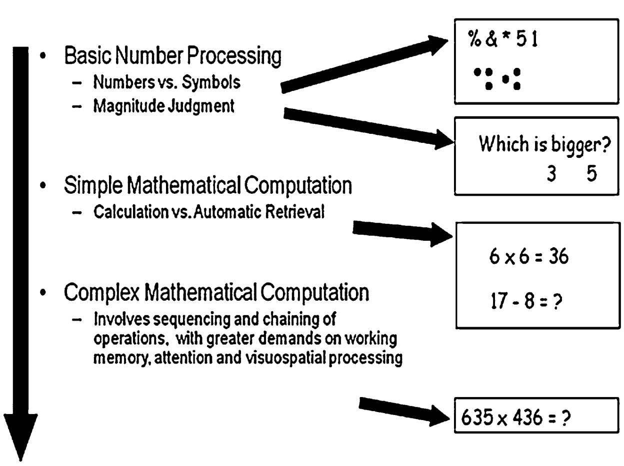

Fig. 1 Levels of information processing in arithmetic

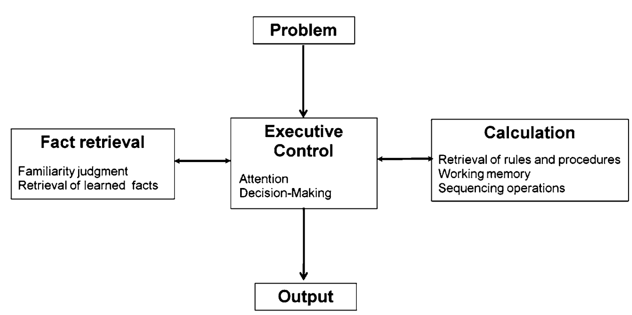

Fig. 2 Cognitive processes involved in arithmetic. Arithmetic problem solving involves multiple cognitive steps, including fact retrieval, associative recall, attention, sequencing, working memory and decision making. The extent to which these processes are engaged varies with prior familiarity with the stimulus and rules that an individual needs to use to make appropriate behaviors. Cognitive neuroscience examines the neural basis of these processes by carefully manipulating one or the other factor

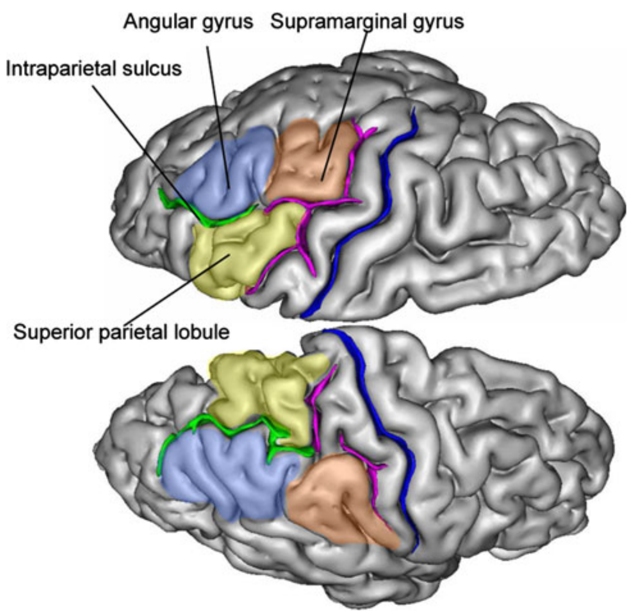

Fig. 3 Neuroanatomy of posterior parietal cortex regions involved in arithmetic. Left and right hemisphere views of posterior parietal cortex regions that are typically activated by mental arithmetic tasks, including the superior parietal lobule (*yellow*), angular gyrus (*blue*), and supramarginal gyrus (*orange*) delineated by the intraparietal sulcus (*green*) and the post central sulcus (*pink*). The central sulcus (*blue*) is included as a reference point. The intraparietal sulcus divides the superior and inferior parietal cortex, which together constitute the posterior parietal cortex. The angular and supramarginal gyri together constitute the inferior parietal cortex (color figure online)

in arithmetic (Fig. 4). These studies have implicated the left and right posterior parietal cortex in number processing and fact retrieval, and the prefrontal cortex in decision making, sequencing, working memory and attention necessary to retrieve learned facts and to perform more elaborate computations when needed. All three major subdivisions of the inferior parietal lobule, comprising the intraparietal sulcus, and supramarginal and angular gyrus (Fig. 3), have been linked to arithmetic processing (Delazer, et al., 2004; Gruber, Indefrey, Steinmetz, & Kleinschmidt, 2001; Lee, 2000; Simon, Mangin, Cohen, Le Bihan, & Dehaene, 2002). Although most studies report robust activation in the posterior parietal cortex during arithmetic processing, these areas are involved in other non-arithmetic and non-numerical operations. Indeed, considerable overlap exists between brain regions that have been implicated and those involved in attention, working memory and lexical and linguistic processes (Chang, Crottaz-Herbette, & Menon, 2007; Crottaz-Herbette, Anagnoson, & Menon, 2004; Crottaz-Herbette & Menon, 2006). Research has shown that similar bilateral posterior parietal cortex regions are activated during various types of mathematical as well as during non-mathematical tasks (Gruber, et al., 2001; Simon, et al., 2002; Venkatraman, Ansari, & Chee, 2005), suggesting that these regions have core functions that the brain uses in the service of arithmetic.

Despite the co-activation of the posterior parietal cortex and the prefrontal cortex in most arithmetic tasks, there is evidence to suggest that their functional roles can be dissociated. In an effort to further delineate the differential roles of the posterior parietal cortex and prefrontal cortex in arithmetic, we examined the effects of cognitive load on arithmetic by varying the number of operands and the rate of stimulus presentation in a factorial design (Menon, Rivera, White, Glover, & Reiss, 2000b). We found quantitative differences in activation of the parietal and prefrontal cortices as well as the recruitment of additional brain regions, including the caudate nucleus and the cerebellum, with increasing task difficulty. More importantly, the main effect of arithmetic complexity was observed in the left and right posterior parietal cortex, while the main effect of domain-general task difficulty was observed in the left inferior frontal cortex. These findings suggest that the posterior parietal cortex plays a more crucial and specific role in arithmetic processing, independent of other processing demands.

Based on these and other findings, researchers have increasingly focused their attention on understanding how the functional organization of the posterior parietal cortex changes with learning. Although we currently understand very little of how expertise for arithmetic develops in children, related research in adults offers a helpful conceptual framework. We analyzed regional differences in brain activation between perfect and imperfect performers and found a relationship between left posterior parietal cortex activity and mental calculation expertise (Menon, et al., 2000a). Perfect performers had an accuracy of 100%

Fig. 4 Brain areas typically activated during arithmetic. Left and right hemisphere and dorsal views of the posterior parietal cortex and prefrontal cortex areas typically activated during arithmetic, compared to number identification tasks (adapted from Menon, et al. 2000b)

and were significantly faster than the rest of the subjects. They had significantly less activation only in the left posterior parietal cortex, a reduction that may be related to functional optimization of performance associated with skill mastery and long-term practice effects. More controlled studies in adults suggest that learning arithmetic is associated with major functional reorganization within the posterior parietal cortex, such that the load on the intraparietal sulcus is reduced and angular gyrus responses are increased relative to baseline (Delazer, et al., 2005; Ischebeck, Zamarian, Egger, Schocke, & Delazer, 2007; Zamarian, Ischebeck, & Delazer, 2009). Grabner and colleagues found increased response in the left angular gyrus when the answer was retrieved, whereas activation in the prefrontal cortex was enhanced when computational procedures were used for problem solving (Grabner, et al., 2009). Similarly, Wu and colleagues found that greater bilateral angular gyrus deactivation was associated with poorer performance (Wu, et al., 2009). However, these differences arise from decreased deactivation, rather than increased activation during retrieval, and the precise role of the angular gyrus in facilitating retrieval remains unclear (Wu, et al., 2009).

Other forms of learning including priming may also contribute to the development of mathematical expertise. Thus, for example, in adults we found that repeated stimulus presentation is associated with widespread decreases in brain response within the frontal, temporal and occipital lobes during mathematical information processing (Salimpoor, Chang, & Menon, 2009). Furthermore, improvements in reaction time were associated with increased recruitment of the hippocampus, the posterior cingulate cortex, precuneus and the adjoining retrosplenial cortex: brain regions that mediate associative encoding and retrieval. It is likely that similar mechanisms may play an important role in the acquisition of arithmetic skills in children as a result of repeated exposure.

3 Neurodevelopmental changes in arithmetic

Normative functional neuroimaging studies have implicated the intraparietal sulcus within the posterior parietal cortex as a region specifically involved in the representation and manipulation of numerical quantity (Dehaene, Piazza, Pinel, & Cohen, 2003). With experience and learning, the intraparietal sulcus builds an increasingly amodal, language-independent semantic representation of numerical quantity (Ansari, 2008; Bruandet, Molko, Cohen, & Dehaene, 2004; Cantlon, Brannon, Carter, & Pelphrey, 2006; Cantlon, et al., 2009; Rosenberg-Lee, Tsang, & Menon, 2009). In addition to the intraparietal sulcus, depending on the nature and complexity of specific tasks, mathematical information processing also critically involves activation and deactivation in a more distributed network of regions within the dorsal visual stream encompassing the superior parietal lobule, the angular and supramarginal gyri in the posterior parietal cortex and the ventral visual stream encompassing the lingual and fusiform gyri in the inferior temporal cortex (Delazer, et al., 2003; Grabner, et al., 2009; Menon, et al., 2000b; Rickard, et al., 2000; Wu, et al., 2009; Zago, et al., 2001).

In one of the first studies of its kind, Rivera and colleagues examined the neural correlates of arithmetic skill development in children using a task involving simple addition and subtraction tasks that were appropriate for 8-year-old participants (Rivera, Reiss, Eckert, & Menon, 2005). Participants viewed arithmetic equations in the form '$a + b = c$' and were asked to judge whether the results were correct or not. During addition and subtraction trials for which accuracy was comparable across age, children showed a pattern of reduced and increased activation compared to adults, suggesting dissimilar trajectories of functional maturation in particular brain regions involved in arithmetic. We found that children had less activation in the left supramarginal gyrus and adjoining intraparietal sulcus, an area that has been consistently implicated in arithmetic processing across a number of lesion and fMRI studies. Increased activation in the left lateral occipital temporal cortex, an area thought to be important for visual word and symbol recognition (Cohen & Dehaene, 2004; Hart, Kraut, Kremen, Soher, & Gordon, 2000; Kronbichler, et al., 2004; Price & Devlin, 2003, 2004), was also observed in adults.

On the other hand, children showed greater activation in the prefrontal cortex, including the dorsolateral and ventrolateral prefrontal cortex as well as in the anterior cingulate cortex (Fig. 5; Table 1). These findings suggest a process of increased functional specialization of the left posterior parietal cortex with age, with decreased dependence on working memory and attentional resources. Additionally, younger children exhibited greater left hippocampal and parahippocampal gyrus activation. Both the hippocampus and the parahippocampal gyrus are known to play a major role in encoding and retrieval of facts (Squire, Stark, & Clark, 2004). It is also likely that the parahippocampal gyrus mediates convergence of high-level input from the visual association cortex into the hippocampus (Suzuki & Amaral, 1994), thereby facilitating the persistence of representations in short-term memory (Eichenbaum, 2000). The greater activation seen in this region in younger subjects may reflect the greater recruitment of processing resources to sustain appropriate memory representations and may also reflect generalized novelty effects. With increased experience and exposure, medial

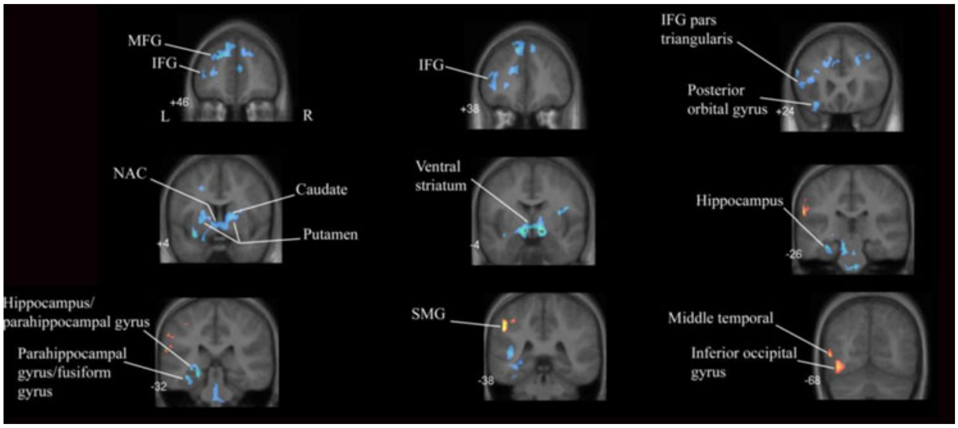

Fig. 5 Neurodevelopmental changes in arithmetic. Compared to adults, children showed greater activation in the prefrontal cortex, basal ganglia and the hippocampus (shown in *blue*) during two operand arithmetic tasks. Adults showed greater activation in the supramarginal gyrus and the lateral occipital cortex (shown in *red*). Task accuracy was matched across the groups. *IFG* inferior frontal gyrus, *MFG* middle frontal gyrus, *NAC* nucleus accumbens, *SMG* supramarginal gyrus (adapted from Rivera, et al., 2005) (color figure online)

Table 1 Summary of neurodevelopmental changes in arithmetic

Putative function	Brain areas	Direction of developmental effects
Number processing	Mid-posterior intraparietal sulcus	No change
Long-term memory: fact retrieval	Supramarginal gyrus Angular gyrus?	Increase with age (children < adults)
Short-term procedural and episodic memory	Basal ganglia hippocampus	Decrease with age (children > adults)
Working memory and decision making	Dorsolateral PFC Ventrolateral PFC	Decrease with age (children > adults)

temporal lobe activations may decrease as stimuli become less novel (Menon, White, Eliez, Glover, & Reiss, 2000c). Children also showed greater activation in the dorsal basal ganglia, including the caudate and putamen. The basal ganglia are known to be critical for procedural memory (Ghilardi, et al., 2000), i.e., memory for procedures and habits, and it plays a supportive role in the maintenance of information in working memory (Chang, et al., 2007). Furthermore, the prefrontal cortex, in concert with medial temporal lobe and dorsal basal ganglia memory systems, regulate declarative, procedural and working memory (Packard & Knowlton, 2002). All of these three regions showed greater activation in children. Parallel increases in hippocampus and basal ganglia activation in children have also been recently reported in a task involving overriding a learned action in favor of a new one (Casey, Thomas, Davidson, Kunz, & Franzen, 2002). These findings provide evidence for greater involvement of and greater reliance on memory functions subserved by the hippocampus and the basal ganglia in children. We have proposed that greater activation of these areas in children reflects more demands on memory during the initial stages of arithmetic learning in children.

Such cross-sectional studies do not, however, adequately capture neurodevelopment processes involved in the formation of long-term memories for arithmetic facts, which arise from the repeated use of counting and other procedures during problem solving. Extensive behavioral research has focused on the cognitive mechanisms that influence strategies employed in solving arithmetic problems (Ackerman, 1996; Geary, Hoard, Byrd-Craven, & DeSoto, 2004; Geary, et al., 2007; Wu, et al., 2008). Efficient fact retrieval is preceded by a period during which children use a mix of counting and other procedures, as well as retrieval to solve arithmetic problems; e.g., counting to solve some problems and retrieving the answer to others (Siegler & Shrager, 1984). As an example, counting "five, six, seven, eight" to solve the problem 5 + 3 results in a long-term memory association between the answer ("8") and the problem ("5 + 3"). After many such episodes, children begin to retrieve the answer when presented with the problem. The neural mechanisms mediating individual differences in children's strategy used during arithmetic problem solving are not fully understood; it is likely that this involves the integration across a distributed brain network involved in higher-order visuospatial processing, memory and cognitive control, as suggested by recent studies on brain mechanisms mediating different arithmetic strategies (Grabner, et al., 2009; Wu, et al., 2009).

4 Neurodevelopmental changes in related cognitive processes

As noted above, the development of arithmetic skills relies on several cognitive processes such as working memory, memory encoding and retrieval, decision making, and attention. Memory processes, such as encoding and retrieval of facts, short-term storage capacity and control processes on the contents of stored memory, all undergo significant developmental changes in children (Kwon, Reiss, & Menon, 2002; Menon, Boyett-Anderson, & Reiss, 2005; Ofen, et al., 2007). We now know that neurodevelopmental changes in these processes are much more protracted than previously believed. Throughout early childhood, an abundance of new synaptic connections among neurons are generated in the brain. Synaptic pruning (removal of unnecessary connections between neurons), myelination of white matter tracts (which increases speed of communication between neurons), maturation of the prefrontal cortex and development of connections to the prefrontal cortex increase in children between ages 6 and 14 years (Lyon & Rumsey, 1996). Prominent, age-related changes in gray matter (which contains neurons and other supporting cells) and white matter (which contains fiber tracts that link multiple brain areas) volumes are evident during childhood and appear to reflect ongoing maturation and remodeling of the central nervous system (Gogtay, et al., 2004; Shaw, et al., 2008; Supekar, Musen, & Menon, 2009). Thus, brain development during these years makes available additional processing resources, thereby facilitating more efficient processing of complex cognitive operations (Kail & Park, 1994).

4.1 Working memory

Behavioral studies have shown that working memory plays an important role in supporting arithmetic. The development of simple arithmetic generally begins with an initial reliance on procedural knowledge such as counting, followed by a gradual shift to retrieval (Ashcraft, 1982). Adults typically retrieve from memory the answer to a simple number problem (e.g., $3 + 4$) through the activation of associative links between number combinations and solutions (Geary, Widaman, & Little, 1986; LeFevre, Bisanz, & Mrkonjic, 1988; Miller, Perlmutter, & Keating, 1984; Rickard & Bourne, 1996). Research on arithmetic performance in children has shown that although there is some reliance on memory retrieval of solutions at the age of 7–8 years (grade 2), counting procedures and other reconstructive strategies are more prominently used (Baroody, 1987; Groen & Parkman, 1972). Working memory is pivotal to many aspects of learning mathematics (Bull, Epsy, & Wiebe, 2008; Geary, 1990; Swanson & Sachse-Lee, 2001; van der Sluis, van der Leij, & de Jong, 2005; Wilson & Swanson, 2001). The central executive plays an important role in sequencing operations, coordinating the flow of information and guiding decision making, particularly when problems are more complex and facts cannot be easily retrieved from memory. For less well-rehearsed operations, such as subtraction and division, working memory is important for holding information in mind and for manipulating intermediate quantities, both of which can be impaired by low working memory capacity (Geary & Brown, 1991; Geary, Hamson, & Hoard, 2000; Geary, et al., 2004; Hitch & McAuley, 1991; Passolunghi & Siegel, 2001, 2004; Siegel & Ryan, 1989; Swanson, 1994; Swanson, Cooney, & Brock, 1993). Poor working memory leads to greater reliance on immature problem-solving strategies in children (Geary & Damon, 2006). Furthermore, performance pressures and anxiety can reduce effective working memory capacity (Beilock & Carr, 2005), leading to further decrements in arithmetic performance in children with mathematical difficulties.

Working memory undergoes significant neurodevelopmental changes during childhood. Increased recruitment of the posterior parietal cortex, prefrontal cortex and basal ganglia over the course of development is associated with better performance in a range of working memory tasks (Bunge & Wright, 2007). Furthermore, posterior parietal cortex and the dorsolateral prefrontal cortex regions that support working memory continue to mature from the age of 7–25 years (Kwon, et al., 2002) and overlap with brain regions that show developmental changes during arithmetic task performance (Rivera, et al., 2005). Mature dorsolateral prefrontal cortex activity is also crucial for facilitating suppression of distracting information (Olesen, Macoveanu, Tegnér, & Klingberg, 2007). Very little is currently known about how these neurodevelopmental changes impact arithmetic problem solving and skill acquisition. Behavioral research suggests that the central executive and phonological loop facilitate performance during early stages of mathematical learning, whereas visuo-spatial representations play an increasingly important role during later stages (Meyer, Salimpoor, Wu, Geary, & Menon, 2009). We suggest that these changes reflect a shift from prefrontal to parietal cortical functions during mathematical skill acquisition. The increased reliance on visuo-spatial representations is consistent with neurocognitive studies that have provided evidence for a shift from reliance on prefrontal cortex functions to those mediated by the posterior parietal cortex with increased mathematical skills. With development (Rivera, et al., 2005), as with extended practice in adults (Ischebeck, et al., 2007; Ischebeck, et al., 2006), there is a shift from central executive processes subserved by the prefrontal cortex to more specialized mechanisms in the posterior parietal cortex.

Since children rely to a greater extent on calculation and working memory to perform arithmetic tasks, this raises the question of whether working memory enhancements can be used to improve skill acquisition. Behavioral studies in adults suggest that arithmetic knowledge can improve with working memory training. For example, Girelli and colleagues (Girelli, Delazer, Semenza, & Denes, 1996) reported that, after working memory training, two patients with a selective deficit on multiplication facts improved on these tasks. Interestingly, recovery and pattern of fact reacquisition mirrored patients' strategy choice in relearning the arithmetic facts. Recent imaging studies have also provided evidence for neural plasticity in working memory tasks after 5 weeks of training (Olesen, Westerberg, & Klingberg, 2004; Westerberg & Klingberg, 2007). Brain areas showing plasticity include the posterior parietal cortex and dorsolateral prefrontal cortex regions that overlap with regions activated during arithmetic tasks. Further studies are needed to understand potential mechanisms by which working memory training can impact speed and accuracy on arithmetic task performance in children, particularly in more complex tasks such as multi-digit computations.

4.2 Memory encoding and retrieval

Children memorize arithmetic facts through repeated exposure and this process engages episodic memory (memory for events) and semantic memory (memory for concepts and facts) systems. While children as young as 3–4 years of age can form episodic memories for pictures (Arterberry, Milburn, Loza, & Willert, 2001), performance on episodic memory tasks (tasks demanding recollection of events) continues to improve until the age of 11, at which point memory abilities begin to resemble those of adults in several respects (Schneider & Goswami, 2002). However, the capacity of memory systems, the speed of retrieval and the strategies used to remember continue to develop through young adulthood (Cycowicz, 2000; Cycowicz, Friedman, Snodgrass, & Duff, 2001).

Despite the relative wealth of knowledge regarding the development of memory abilities, little is known about the neural organization of memory in children and adolescents. In adults, a wide range of electrophysiological, lesion and neuroimaging studies have shown the critical involvement of the medial temporal lobe, including the hippocampal region, in memory encoding (Schacter & Wagner, 1999). The development and maturation of hippocampus and other brain regions involved in memory encoding are, however, poorly understood. There is evidence, however, to suggest that increased functional interactions between the medial temporal lobe and the prefrontal cortex may underlie the development of more effective memory-encoding strategies (Menon, et al., 2005). Whether similar processes contribute to arithmetic learning in children remains to be investigated.

4.3 Interference resolution and decision making

In addition to memory retrieval and computation, solving an arithmetic question involves decision making at several levels. For example, in multiple-choice testing formats, children should be able to judge the accuracy of the correct answer as well as inhibit distracting and incorrect choices. These decision-making processes can affect the speed and accuracy with which individuals respond to arithmetic problems (Kail & Salthouse, 1994). Difficulty in inhibiting incorrect or irrelevant associations is one proposed explanation for arithmetic deficits in some children (Geary & Damon, 2006). Further, children may manifest reduced confidence in assessing the accuracy of a retrieved fact. The extent to which such decision-making strategies contribute to slower and more error-prone performance in children is unclear. However, findings from brain imaging studies in adults may shed light on brain mechanisms mediating interference resolution and decision making.

One approach to studying decision making in the context of mental arithmetic is to use a Stroop-like interference paradigm in which participants are presented with incorrect answers to arithmetic problems. Investigations of the psychological and neural bases of arithmetic reasoning are frequently based on verification tasks in which subjects are presented with equations of the form, "$2 + 3 = 5$", and are asked to make a decision regarding whether the presented answer is correct or incorrect (Menon, Mackenzie, Rivera, & Reiss, 2002). A key aspect of this type of arithmetic reasoning is the ability to distinguish between incorrect and correct arithmetic equations. Electrophysiological studies in humans have demonstrated that processing incorrect arithmetic equations elicits a prominent "N400" event-related potential compared to processing correct equations (Niedeggen, Rosler, & Jost, 1999). We investigated the neural substrates of this process using event-related analysis (Menon, et al., 2002). Subjects were presented with arithmetic equations and asked to indicate whether the solution displayed was correct or incorrect. The left dorsolateral prefrontal cortex and the left ventrolateral prefrontal cortex showed greater activation to incorrect, compared to correct, equations. These results provide the first brain imaging evidence for differential processing of incorrect versus correct equations. The prefrontal cortex activation observed in processing incorrect equations overlaps with brain areas known to be involved in working memory and interference processing.

The dorsolateral prefrontal cortex region differentially activated by incorrect equations was also involved in overall arithmetic processing, whereas the ventrolateral prefrontal cortex was activated only during the differential processing of incorrect equations. Differential response to correct and incorrect arithmetic equations was not observed in the posterior parietal cortex regions, which have been shown to play a critical role in mental arithmetic in several previous studies. The pattern of brain response observed is consistent with the hypothesis that processing incorrect equations involves detection and resolution of the interference between the internally computed and externally presented incorrect answer. More specifically, greater activation during processing of incorrect equations appears to reflect additional operations involved in maintaining the results in working memory, while subjects attempt to resolve the conflict and select a response. These findings allowed us to further delineate and dissociate the contributions of prefrontal and parietal cortices to arithmetic reasoning. Importantly, findings from this study provide insights into the prefrontal cortex mechanisms underlying decision making in arithmetic. Because the prefrontal cortex matures relatively slowly compared to the posterior parietal cortex, children may be slower or have particular difficulties with certain types of arithmetic problems that require reasoning and interference resolution even when computational and retrieval skills are mature. Further research is needed to better understand the role of these processes in the development of problem-solving abilities in typically developing children as well as in children with specific learning disabilities.

5 Implications for learning and academic achievement

Mathematical learning in children is dependent on the development of several component processes. Some of these processes, especially decision making and working memory, have protracted developmental time lines (Kwon, et al., 2002). Although the current literature is limited in terms of disentangling the neurodevelopmental aspects of specific cognitive processes in relation to their impact on arithmetic skill development, research in adults has provided valuable information on brain systems that mediate optimal task performance and learning in this domain. In essence, the literature reviewed above suggests that with development, as with extended practice, there is a shift from prefrontal cortex-mediated information processing to more specialized mechanisms in the posterior parietal cortex. It is likely that conceptual and procedural knowledge can both contribute to these changes, thereby improving processing efficiency. Although the degree to which the two types of knowledge contribute to stable changes in brain response is not well understood, freeing the prefrontal cortex from computational load and thus making available valuable processing resources for more complex problem solving and reasoning is a key factor in promoting mathematical learning and skill acquisition (van Merrienboer & Sweller, 2005). In this vein, teaching strategies that emphasize repeated performance, leading to more automatized retrieval, may be beneficial for creating core knowledge in a way that minimizes processing load on the prefrontal cortex.

Cognitive neuroscience research has initially focused on dissociating cognitive and neural processes involved in mental arithmetic by manipulating the surface format and complexity of computations. More recently, research has begun to focus on the neural basis of arithmetic skill development, including the effects of practice and learning (Delazer, et al., 2004; Delazer, et al., 2005; Ischebeck, et al., 2007; Ischebeck, et al., 2006). Critically, these studies have underscored the crucial role of functional changes in the posterior parietal cortex that facilitate more efficient arithmetic task performance. To develop optimal learning paradigms, appropriate randomized control studies in children are necessary at this time. Here, studies based on integrating experimental paradigms developed in cognitive neuroscience studies of arithmetic with insights from educational practice (Fuchs, et al., 2005; Fuchs, et al., 2007) offer the best hope for developing a more complete understanding of the mechanisms underlying arithmetic skill development. As noted above, cognitive neuroscience has also brought to the forefront the role of working memory, episodic and semantic memory, as well as decision-making and attentional processes in both accurate task performance and in facilitating the maturation and development of arithmetic skills. To relate neuroscience to educational practice, future studies will need to integrate the neuroscience of core cognitive processes outlined above with rigorous methods for learning and remediation developed by educational psychologists.

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