Incident Photon Direction Calculation Using Bayesian Estimation for High Energy Photon Detector Systems with 3D Positioning Capability

Angela M K Foudray, *Student Member, IEEE,* Garry Chinn, *Member, IEEE,* and Craig S Levin, *Member, IEEE*

***Abstract*— We are developing high resolution PET systems comprising detectors that can position photon interactions in three dimensions and distinguish between individual interactions for single photons entering the system. Having these interaction locations and energies, algorithms can be employed to gain more information about the photon before interaction, including the incident direction and energy. We have developed a maximum likelihood algorithm using Bayesian methods to estimate a single photon's incident angle and interaction location, provided that it produced more than one detectable interaction. Both the phi (ϕ) and theta (θ) angle are calculated in this algorithm. The forward model was simulated using Geant4 Application for Tomographic Emission (GATE). The probabilities were calculated for a system comprised of 1mm x 1mm x 3mm LSO arrays coupled to position-sensitive avalanche photodiodes with 12% energy and 2ns coincidence time resolution. A training set of $\sim$20,000 single photon events were used per angle to calculate the likelihood and prior probabilities for all angles in the range detected by the system (nearly 180 degrees in theta and phi). A test set of singles for particular angles produced an incoming photon angular estimation with a $\sim$16 degrees root mean squared (RMS) deviation in both θ and ϕ.**

***Index Terms*— 3D detectors, Compton, event filtering, angular resolution, Bayes**

I. Introduction

We are developing emission tomography systems that are being designed to acquire precise positioning data both in the transaxial direction, as well as radial direction (depth of interaction (DOI) direction). These systems will have the ability to distinguish the individual interactions in the detector, e.g. Compton, Rayleigh, photoelectric, etc. These interactions provide insight about the incident photon's initial direction and energy using techniques such as Compton Kinematics (CK) or Maximum Likelihood (ML) algorithms.

Having information about the characteristics of an incoming photon could allow a higher degree of certainty about how that photon should be used, if at all. Benefits of having direction and energy information about a photon incident on the detectors include: more precise event typing (Singles, Randoms, Scatters, Multiples, Trues) for 1) filters, 2) reclamation of previously unusable events and 3) for LOR assignment (event positioning). For instance; knowing the incident direction for two out of three singles that have occurred in a system's acquisition coincidence time window may eliminate one of the photons as belonging to a coincidence, regaining a true event. This is particularly germane for detector systems based on photodetectors with relatively poor time resolution, such as Cadmium Zinc Telluride (CZT). Systems with poor time resolution may suffer signal-to-noise ratio loss due to large time windows, resulting in a sizable fraction of multiples, which are currently discarded events.

Previous analysis of similar detectors for gamma-ray tracking using Compton Kinematics [1] gives an estimation of spatial and energy resolution necessary for adequate calculation of the incident angle. Since the photodetectors that comprise the system under study are position-sensitive avalanche photodiodes (PSAPDs), device centroiding and inter-PSAPD gaps larger than the size of the segmented lutetium oxyorthosilicate (LSO) crystals will degrade previous CK estimations.

This work is instead developing a Maximum Likelihood algorithm that includes a realistic model of the detector system (energy resolution, position resolution, device centroiding and crystal gaps) using Bayesian methods for extracting a single photon's incident angle with respect to the detector.

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Angela MK Foudray is a student at the University of California-San Diego, Department of Physics, La Jolla, CA, USA and is working in the Department of Radiology, the Molecular Imaging Program at Stanford University (email: afoudray@stanford.edu).

Garry Chinn and Craig S Levin are in the Department of Radiology and the Molecular Imaging Program at Stanford University, Palo Alto, CA, USA (email: gchinn@stanford.edu and cslevin@stanford.edu respectively).

II. Theory

The ML technique employed in this study uses Bayes' theorem [2], which estimates the value of a variable, given a particular type of input information. In the case of this study, we want to find the probability that a photon entered the detector system from a particular angle, knowing the signals that are reported from the detectors in the system, i.e. knowing the position in x, y and z, as well as the energy deposited in each detector involved in one photon event. To perform the survey, the detector system needs to be probed at all the possible incident angles (θ,ϕ) independently, and look at the resulting sets of (x,y,z,E) data. A histogram could then be obtained, or probabilty map, with dimensions equal to the number of variables we observe. This histogram, is the prior probability map. The Bayes' method relates the prior probabilities (interaction locations and energies) to the posterior probabilities (indicent angle) in the following way:

$$P(\Theta|x_1,x_2) = \frac{P(x_1,x_2|\Theta) * P(\Theta)}{P(x_1,x_2)} \quad (1)$$

where (x_1,y_1,z_1,E_1) and (x_2,y_2,z_2,E_2) have been abbreviated x_1 and x_2 respectively, (θ,ϕ) has been abbreviated Θ, and for example $P(x_1, x_2|\Theta)$ is the probability of obtaining x_1 and x_2 given Θ. Equation 1 can be broken down into:

$$P(x_1, x_2) = P(x_1) * P(x_2|x_1) \quad (2)$$

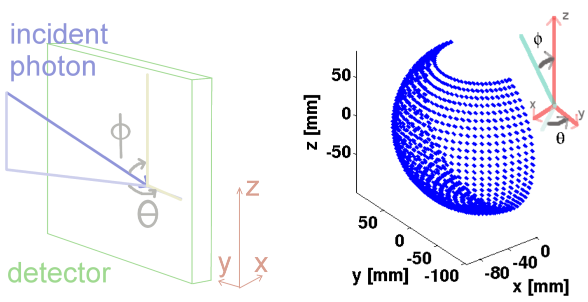
which becomes

$$P(\Theta|x_1, x_2) = \frac{P(x_1|\Theta) * P(x_2|x_1, \Theta) * P(\Theta)}{P(x_1) * P(x_2|x_1)} \quad (3)$$

In other words, knowing the probability of getting a particular x_1 and x_2 result from a particular incident Θ, for all Θ, we can know the probability of getting a particular Θ from knowing x_1 and x_2 and their conditional probabilites, which is exactly what we have gathered in the training set and what we want from the calculating (3).

III. Methods

The probabilities of (3) were obtained using the Monte Carlo package GATE (Geant4 Application for Tomographic Emission) for the forward model. Two sets of interaction locations and energies for each location and angle studied were generated - one for the training set (likelihood and prior calculations, right hand side of (3)) and one for the test set (posterior probabilities, left hand side of (3)).



(a) (b)

Fig. 1. Illustration of coordinate system and figure displaying the source locations in (x,y,z) of GATE system space.

Specifically, a pencil beam was directed into the detector system from a point source for every crystal location, and at every ϕ and θ angle value in the range detected by the system (nearly π radians - see Fig. 1). Due to the symmetries of the system, this was only necessary for one side of the crystals in a module.

In processing this data, to limit the space required to bin the interactions and still obtain high localization of interactions, a reduced space was used where

$$\alpha' = \alpha - COM_\alpha \quad (4)$$

where α = x,y or z and COM_α is the center of mass in the α coordinate.

In order to calculate (3), probability density functions (PDFs) in (x',y',z',E) space were calculated along with the conditional PDFs for each (x_1, x_2) pair for each Θ. In order to determine the algorithm's ability to predict incident angle only knowing the interaction locations, test sets of known incident angle data was used to determine the point spread functions in (θ,ϕ) space. The test sets had ~400 events for each (θ,ϕ) source location in the study. The root mean squared deviation from the angle bin with maximum counts was used to calculate the angular resolution of the algorithm.

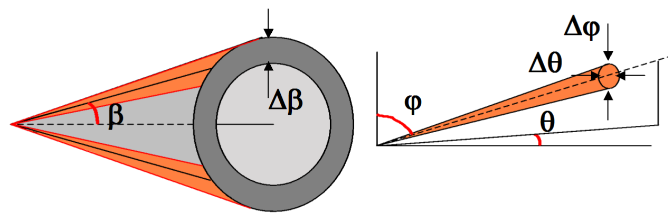


Fig. 2. Comparison of the meaning of "resolution" for the Left: Compton Kinematics method of angle determination, and Right: this Bayesian method's angle determination.

IV. Results and Discussion

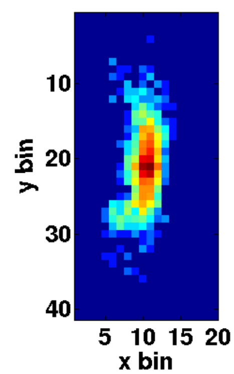


Fig. 3. Slice (z' bin = 0, E bin = 1 (0-250 keV), for all x', y') of the probability distribution function for θ=10 and ϕ=12. The histogram shown is the log of the counts in each of the x' and y' bins.

Posterior and conditional probability maps were determined for each event generated using 20,000 training events for each (θ,ϕ). Figure 3 shows the logrithm of the counts in a slice in x'-y' space of the $P(x_1|\Theta)$ 4-dimensional probability map in (3).

The probability maps were used to calculate the left hand side of (3), or the probability of a particular incident (θ,ϕ). This was done for all incident angle bins (θ,ϕ) and the incident angle bin with the maximal probability was chosen for each event in the test set. The results were then histogrammed. One such histogram is shown in Fig 4 - this is the point spread function in angular space for the algorithm.

Using 20,000 training events lead to a sparsly populated probability matrix, giving only on average 1/10th if an event per (x',y',z',E) bin, but the bulk of the bins contained no counts at all. More training counts would likely lead to higher statistical certainty of the relative probabilities of each bin and therefore possibly more precise angle determination, however the point spread function for the tested (θ, ϕ) were already relatively compact - see Fig. 4.

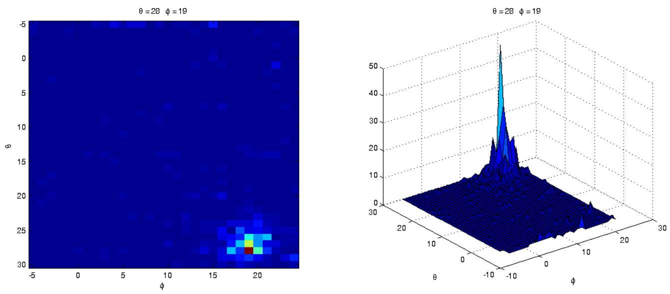


Fig. 4. Angular point spread function for a typical incident angle, shown: (θ,ϕ)=(28,19). Left: Projection of 2D histogram to best see the location of the peak, Right: Side view of 2D histogram to visualize distribution.

A profile through the point spread function histogram along both the θ and ϕ directions was used to determine the angular resolution for each (θ,ϕ) tested. The RMS deviation from the angle bin with the maximum counts - which incidently was always the correct (θ, ϕ) bin - can be seen in Fig. 5 for each angle sampled in the space. The RMS deviation in angle for the θ direction was $15.2^o \pm 5^o$ and was $15.7^o \pm 7^o$ for the ϕ direction. It should be reiterated that the deviation of the RMS values as well as the RMS values themselves would likely decrease with an increase in training data statistics, though it has not been studied here.

V. Conclusions and Future Work

A maximum likelihood algorithm has been developed to estimate a photon's incident direction for photons which have more than one detectable interaction in the detector. The algorithm calculates both the incident θ and ϕ angle, with resolution $<16^o$ in both directions, limiting the incoming direction to a narrow cone. Determining incident angle and position can be used for filtering and/or including previously rejected events, increasing signal-to-noise and contrast in a reconstructed image.

References

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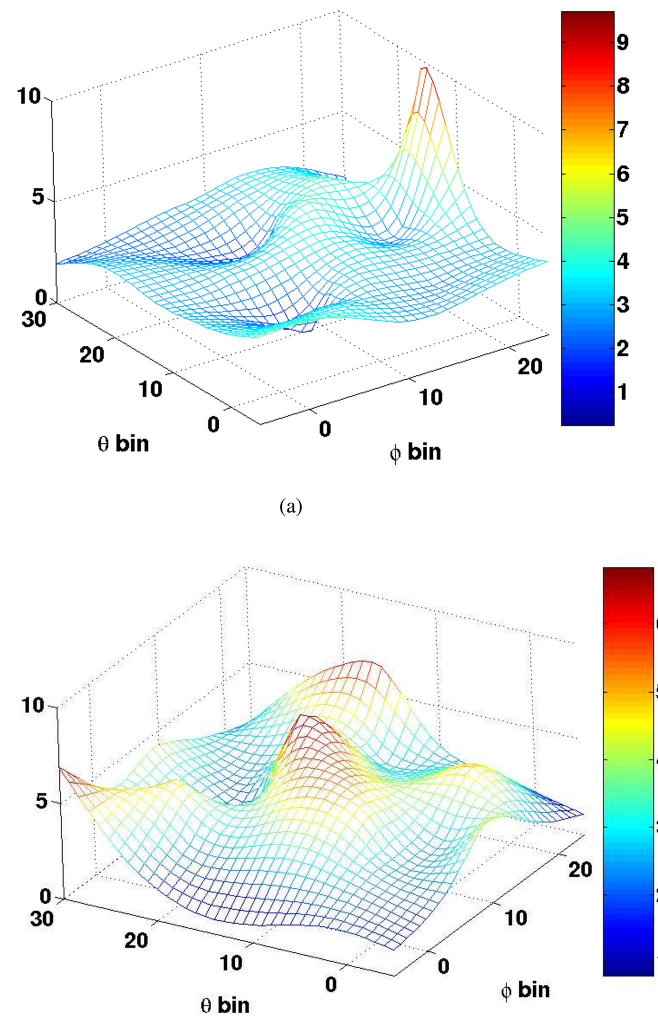
[2] D. J. C. MacKay, *Information Theory, Inference, and Learning Algorithms*, version 7.2, 4th ed. Cambridge University Press, 2005.



(a)

(b)

Fig. 5. Root mean squared variance from the true angle in both the θ and ϕ direction the for each (θ,ϕ) bin. Top: ϕ RMS deviation, Bottom: θ RMS deviation. Variance is measured in bins (5^o in θ and ϕ).