How Do the Accrual Pattern and Follow-Up Duration Affect the Hazard Ratio Estimate When the Proportional Hazards Assumption Is Violated?

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Time-to-event outcomes, such as overall survival and progression-free survival, are often selected as key endpoints in comparative clinical studies. When analyzing results from these studies, estimating the magnitude of the between-group difference is a fundamental step. This is especially important in clinical trials that compare treatments, because quantitative information about the treatment effect is necessary to balance benefits/risks and inform clinical decision making. Conventionally, many comparative oncology studies use the hazard ratio (HR) and a corresponding 95% confidence interval to report the magnitude of the between-group difference, despite a number of important limitations [1–6].

One well-known limitation of HR is the need for a proportional hazards (PH) assumption [1–6]. Specifically, the ratio of the hazard functions for the two groups must be constant over time. Although this limitation is well known, many papers report the HR even when the PH assumption is not met [7–9]. Arguing in favor of using the standard Cox’s method [10] and reporting the HR in all circumstances, investigators assert that it represents an approximate average of the ratio of two hazard functions [11, 12]. However, this is a misunderstanding, because study-specific parameters that should have no bearing on the between-group difference can affect the hazard ratio estimation. For example, two clinical trials evaluating the same intervention and conducted on the same population could produce different HR estimates in a non-PH scenario, no matter how many subjects enroll, just because the duration of follow-up time or the accrual profile differ. Consequently, when the HR is used to summarize the between-group difference using the standard Cox’s method [10] in non-PH scenarios, it is not obvious what “average” quantity the HR is estimating; the resulting HR estimate is clinically uninterpretable and may mislead clinical decision [1, 2]. Our goals were to demonstrate this critical limitation using simulated patient data for PH and non-PH scenarios and to illustrate this limitation by using results from a recent clinical study.

**Numerical Studies**

First, we described two patterns of between-group differences common to comparative oncology clinical trials. The first pattern was PH, with a constant HR of 0.78. Figure 1A shows the survival functions of this pattern (top) and corresponding hazard functions (middle). Figure 1A (bottom) shows the ratio of the hazard functions for the two groups (i.e., time-specific HR). The second pattern was non-PH. Survival functions and hazard functions by group, and their corresponding time-specific HRs, are depicted in Figure 1B (top, middle, and bottom, respectively). This pattern (i.e., survival curves are separated for early time points and then the curves overlap later) is one of the common non-PH scenarios in cancer clinical trials (e.g., Paridaens et al. [7] and Wilke et al. [8]).

Second, we identified two study parameters that should not affect the treatment effect estimate: the patient accrual profile and the follow-up time. In clinical trials, patient enrollment does not necessarily follow a consistent pattern. Sometimes, accrual takes only a few months, but other times, accrual takes much longer. Also, the duration of follow-up after accrual may vary from study to study. Figure 2 illustrates several accrual and follow-up patterns. The y-axis indicates the total number of patients enrolled, and the x-axis indicates calendar time. N denotes the total sample size of the study. T1 and T2 indicate the lengths of the accrual period and the postaccrual follow-up period, respectively. Therefore, T1 + T2 denotes the length of the total study time (i.e., time from the first patient enrolled to the end of trial). Note that in typical event-driven trials, the end of trial is determined by the time when a required number of events are observed and is therefore not a fixed time point.

For our numerical studies, four combinations of T1 and T2 were considered: (a) immediate enrollment, (b) short accrual, (c) intermediate accrual, and (d) long accrual compared with follow-up time (Fig. 2). Specifically, (T1, T2) was...
Note that we had the immediate enrollment pattern (a) as a reference, although it is not realistic to enroll all subjects simultaneously. Combining these four patterns of accrual and follow-up with the two patterns of between-group differences (i.e., PH and non-PH), eight clinical trial scenarios were defined. All scenarios assumed a uniform enrollment rate throughout the given accrual time, except the immediate enrollment case.

The objective of our numerical studies was to determine what population quantity (i.e., HR) the standard Cox’s inference procedure [10] estimates, and to demonstrate how the HR estimate depends on study-specific parameters (i.e., the accrual and/or follow-up time). Using a substantial number of data points for each scenario allowed us to be confident that observed differences in the computed HR across different scenarios were not due to sampling variability. Specifically, for each scenario, we generated 2,000,000 observations (i.e., 1,000,000 simulated observations for each treatment group). The number of events for each of the eight scenarios was at least 773,000, making the length of the 0.95 confidence interval for the HR no greater than 0.006 for all scenarios. This allowed us to compare the computed HRs across different scenarios without regard to the sampling variability.

For each subject, we generated time from the study activation to the enrollment and time from the enrollment to event occurrence. The former was generated from the uniform distribution as described in Figure 2, and the latter was generated from the event time distribution shown in Figure 1 (top). Event time was censored at the end of the follow-up period \(T_1 + T_2\) or the timing of the final data analysis (see supplemental online Data A for details of the data generation procedure). With these simulated data, the HR between two groups was calculated using the Cox procedure. After calculating HR estimates for all scenarios, we looked for variation in these estimates across the four PH scenarios and across the four non-PH scenarios. We reasoned that differences in \(T_1\) and \(T_2\) should not translate into differences in HR estimates.

Results from all eight scenarios appear in Table 1. In PH scenarios, all estimated HRs were the same as the true HR.
of 0.78. In other words, different accrual and follow-up patterns did not yield different HR estimates (Table 1). However, in non-PH scenarios, HR estimates for the four accrual and follow-up patterns were 0.78, 0.73, 0.67, and 0.56, respectively (Table 1). So each HR estimate was not a simple average of the time-specific HR (Fig. 1B, bottom) but rather depended on study-specific parameters that should have had no impact on the treatment effect estimate. The amount of variation in the HR estimates for the non-PH scenarios was not negligible, ranging from 0.56 to 0.78. Arguably, these differences are of sufficient magnitude to affect clinical decision making. If treatment plans were created from these HR estimates, then study parameters unrelated to the actual treatment benefit could have affected clinical decision making.

Next, we examined the magnitude of variation in the HR estimate using data from a recent clinical trial for immunotherapy—a study that evaluated the efficacy and safety of nivolumab versus docetaxel in patients with advanced nonsquamous non-small-cell lung cancer [9]. The top chart in Figure 3 shows the Kaplan-Meier curves of overall survival by group; the reported HR was 0.73 [9]. As is often seen in immunotherapy trials, the survival curves present a delayed difference pattern. The bottom chart in Figure 3 plots the corresponding time-specific HR [13], which was generated by individual-level data that we reconstructed by scanning the reported Kaplan-Meier curves [9, 14]. The HR appears to fluctuate over time, suggesting a departure from the PH assumption. We conducted a similar numerical study generating 2,000,000 data points based on

![Figure 2. Four patterns of patient accrual and length of additional follow-up. Panels (A) through (D) correspond to patterns (a)–(d) in Table 1. Abbreviations: N, total number of patients enrolled; T1, accrual period; T2, additional follow-up period after the end of the accrual.](image-url)

### Table 1. Hazard ratio estimated by Cox’s procedure with eight scenarios

<table>
<thead>
<tr>
<th>Accrual and follow-up pattern (T1, T2)</th>
<th>(a) (0, 40)</th>
<th>(b) (8, 32)</th>
<th>(c) (16, 24)</th>
<th>(d) (32, 8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PH difference</td>
<td>0.78</td>
<td>0.78</td>
<td>0.78</td>
<td>0.78</td>
</tr>
<tr>
<td>Non-PH difference</td>
<td>0.78</td>
<td>0.73</td>
<td>0.67</td>
<td>0.56</td>
</tr>
</tbody>
</table>

T1 + T2 = total study duration.

Abbreviations: PH, proportional hazards; T1, accrual period; T2, additional follow-up period after the end of the accrual.
the estimated survival functions presented in Figure 3 (top) (see supplemental online Data B for details) to demonstrate the extent to which the HR estimate could have varied if the accrual period ($T_1$) or additional follow-up period ($T_2$) had differed. Figure 4 shows the estimated HR with various combinations of ($T_1$, $T_2$). In our numerical study, each scenario has the same total study duration ($T_1 + T_2$) for simplicity. The total study duration ($T_1 + T_2$) could vary in practice, because the study termination is usually not fixed but rather occurs when a prespecified number of events are observed. We found that changes to $T_1$ or $T_2$ could have produced a HR anywhere in the range from 0.73 to 0.81 (Figure 4). Note that the HR variation we observed in this numerical study was a function of the ($T_1$, $T_2$) combinations we chose, where we fluctuated ($T_1$, $T_2$), forcing $T_1 + T_2$ to be a constant. If ($T_1$, $T_2$) varied more than we fluctuated, then the HR range could have been even larger.

**DISCUSSION**

We demonstrated that the HR estimated by Cox’s procedure [10] is affected by two study-specific parameters (i.e., accrual pattern and follow-up time) in non-PH scenarios. Other factors that are possibly independent of the treatment difference (e.g., random dropout rate, withdrawal from the study) will also affect the HR estimate. We have examined two non-PH cases commonly seen in cancer clinical trials. One is the pattern in which survival curves are separated during early time points and then glued together later on [7, 8]; another

**Figure 3.** Overall survival by group with reconstructed data of the CheckMate 057 study. Kaplan-Meier curves (top) and estimated time-specific hazard ratio (bottom).

**Figure 4.** Hazard ratio estimated by Cox’s procedure as a summary of the difference between two survival time distributions presented in Figure 3 with various scenarios of accrual period ($T_1$) and additional follow-up period after the end of accrual ($T_2$), where $T_1 + T_2$ is the total study duration.
is the pattern of delayed separation, which is typical for immunotherapy trials [9]. The variation in the estimated HR was not negligible for both non-PH patterns.

Although no statistical method can confirm that a between-group difference is indeed a PH, statistical testing can help detect a violation of the PH assumption when an investigator ultimately wants to report the HR. For example, testing based on cumulative sums of martingale residual [15] can be implemented easily using the ASSESS statement in SAS’s PHREG procedure, stphtest command in Stata, gof package in R, and so on.

Even when the difference between two groups is a PH, there are other limitations to the HR [1]. For example, it can be challenging to determine whether the observed HR indicates a clinically meaningful benefit (or harm), because there is no reference value from the control group. Given that, careful considerations should be made at the design stage in specifying a primary summary measure. Choosing a statistically robust and clinically interpretable summary measure is critically important to the clinicians who must interpret comparative clinical trials and make treatment recommendations to patients. Unless there is strong justification for using HR as a summary for the treatment effect, it would not be recommended. We recommend using alternative measures that do not rely on any modeling assumptions, such as the difference or ratio of median survival times, $t$-year event rates, and restricted mean survival times [1–4, 6, 16–19].

In summary, the HR estimate from the Cox procedure cannot be interpreted as a simple or meaningful weighted average of the time-specific HR in non-PH cases. Decisions based on HR estimates may be misleading when the PH assumption is violated. Caution should be exercised when investigators report, and readers interpret, HR as the magnitude of the between-group difference.

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**Disclosures**

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**References**